FACTORING TRINOMIALS OF THE FORM $x^2 + bx + c$, WHERE $c < 0$

- Before doing this exercise, you may want to study:
  Basic Concepts Involved in Factoring Trinomials
  Factoring Trinomials of the form $x^2 + bx + c$, where $c > 0$

(more mathematical cats)

Here, you will practice factoring trinomials of the form $x^2 + bx + c$,
where b and c are integers, and $c < 0$.
That is, ***the constant term is negative***.

Recall that the ***integers*** are: $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

As discussed in Basic Concepts Involved in Factoring Trinomials,
you must first find two numbers that add to b and that multiply to c, since then:

$$x^2 + bx + c \;=\; x^2 + (\overbrace{f+g}^{=b})x + \overbrace{fg}^{=c} \;=\; (x+f)(x+g)$$

Since c is negative in this exercise, one number will be positive, and the other will be negative.
(How can two numbers multiply to give a negative result? One must be positive, and the other negative.)
That is, the numbers will have different signs.

When you add numbers that have different signs, then ***in your head you actually do a subtraction problem***.
For example, to mentally add $(-5) + 3$, in your head you would compute $5 - 3$,
and then assign a negative sign to your answer.
Think of it this way: Start at zero on a number line. Walk 5 units to the left, and 3 units to the right.
You end up at -2.
You walked farther to the left than you did to the right, so your final answer is negative.

The sign of b (the coefficient of the x term) determines which number will be positive, and which will be negative:
If $b > 0$, then the bigger number (the one farthest from zero) will be positive.
If $b < 0$, then the bigger number (the one farthest from zero) will be negative.
In other words, the ***biggest*** number takes the sign (plus or minus) of b.

These results are summarized below:

FACTORING TRINOMIALS OF THE FORM $x^2 + bx + c\,,\quad c < 0$

- Check that the coefficient of the square term is 1 .
- Check that the constant term (c) is negative.
- ***It's easier to do mental computations involving only positive numbers.*** So, you will initially ignore all minus signs and just work with the numbers $|b|$ and $|c|$.
- Find two numbers whose DIFFERENCE is $|b|$ and whose PRODUCT is $|c|$.
  That is, find two numbers that ***subtract*** to give $|b|$ and that ***multiply*** to give $|c|$.
- Now (and only now), you'll use the actual plus-or-minus sign of b .
  If $b > 0$, then the bigger of your two numbers is positive; the other is negative.
  If $b < 0$, then the bigger of your two numbers is negative; the other is positive.
  That is, the ***biggest*** number takes the sign (plus or minus) of b .
- Use these two numbers to factor the trinomial, as illustrated in the examples below.
- Be sure to check your answer using FOIL.

EXAMPLES:

Question: Factor: $x^2 + 5x - 6$
Solution: Thought process:
Is the coefficient of the x^2 term equal to 1 ? Check!
Is the constant term negative? Check!
Find two numbers whose difference is 5 and whose product is 6 .
That is, find two numbers that subtract to give 5 and that multiply to give 6 .
The numbers 1 and 6 work, since $6 - 1 = 5$ and $6 \cdot 1 = 6$.
Since the coefficient of x is positive, the bigger number (6) will be positive, and the other will be negative.
The desired numbers are therefore 6 and -1 .
Then,

$$x^2 + 5x - 6 \quad = \quad x^2 + (\overbrace{6 + (-1)}^{=5})x + \overbrace{6 \cdot (-1)}^{=-6} \quad = \quad (x+6)(x-1)$$

Check: $(x+6)(x-1) = x^2 - x + 6x - 6 = x^2 + 5x - 6$

Question: Factor: $x^2 - 5x - 6$

Solution: Thought process:
Is the coefficient of the x^2 term equal to 1? Check!
Is the constant term negative? Check!
Find two numbers whose difference is 5 and whose product is 6.
That is, find two numbers that subtract to give 5 and that multiply to give 6.
The numbers 1 and 6 work, since $6 - 1 = 5$ and $6 \cdot 1 = 6$.
Since the coefficient of x is negative, the bigger number (6) will be negative, and the other will be positive.
The desired numbers are therefore -6 and 1.
Then,

$$x^2 - 5x - 6 \;=\; x^2 + (\overbrace{(-6) + 1}^{=-5})x + \overbrace{(-6) \cdot 1}^{=-6} \;=\; (x - 6)(x + 1)$$

Check: $(x - 6)(x + 1) = x^2 + x - 6x - 6 = x^2 - 5x - 6$

Question: Factor: $x^2 + x - 1$

Solution:
There are no integers whose difference and product are both 1.
Thus, $x^2 + x - 1$ is not factorable over the integers.